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DOES THE INERTIA OF A BODY DEPEND UPON ITS ENERGY CONTENT?

by A. Einstein

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The results of an electrodynamic investigation published by me recently in this journal[1] lead to a very interesting conclusion, which shall be derived here.

There I based myself upon the Maxwell-Hertz equations for empty space along with Maxwell's expression for the electromagnetic energy of space, and also on the following principle:

The laws governing the changes of state of physical systems do not depend on which one of two coordinate systems moving in uniform parallel translation relative to each other these changes of state are referred to (principle of relativity).

Based on these fundamental principles[2], I derived the following result, among others (*loc. cit.*, §8):

Let a system of plane waves of light, referred to the coordinate system (x,y,z), possess the energy ℓ; let the direction of the ray (the wave normal) form the angle φ with the x-axis of the system. If we introduce a new coordinate system (ξ,η,ζ), which is uniformly parallel-translated with respect to the system (x,y,z), and whose origin is moving along the x-axis with velocity v, then the above-mentioned quantity of light—measured in the system (ξ,η,ζ)—possesses the energy

$$\ell^* = \ell \frac{1 - \frac{v}{V} \cos \varphi}{\sqrt{1 - \left[\frac{v}{V}\right]^2}} ,$$

where V denotes the velocity of light. We will make use of this result in the following.

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[1] [1]A. Einstein, *Ann. d. Phys.* 17 (1905): 891.

[2]The principle of the constancy of the velocity of light used there is of course contained in Maxwell's equations.

from The Collected Works of Albert Einstein, Vol. 2: The Swiss Years: Writings 1900-1909, Princeton University Press, 1990. English Translation supplement.

Let there be a body at rest in the system (x,y,z), whose energy, referred to the system (x,y,z), is E_0. The energy of the body with respect to the system (ξ,η,ζ), which is moving with velocity v as above, shall be H_0.

Let this body simultaneously emit plane waves of light of energy $L/2$ (measured relative to (x,y,z)) in a direction forming an angle φ with the x-axis and an equal amount of light in the opposite direction. All the while, the body shall stay at rest with respect to the system (x,y,z). This process must satisfy the energy principle, and this must be true (according to the principle of relativity) with respect to both coordinate systems. If E_1 and H_1 denote the energy of the body after the emission of light, as measured relative to the system (x,y,z) and (ξ,η,ζ), respectively, we obtain, using the relation indicated above,

$$E_0 = E_1 + \left[\frac{L}{2} + \frac{L}{2}\right],$$

$$H_0 = H_1 + \left[\frac{L}{2}\,\frac{1 - \frac{v}{V}\cos\varphi}{\sqrt{1 - \left[\frac{v}{V}\right]^2}} + \frac{L}{2}\,\frac{1 + \frac{v}{V}\cos\varphi}{\sqrt{1 - \left[\frac{v}{V}\right]^2}}\right] = H_1 + \frac{L}{\sqrt{1 - \left[\frac{v}{V}\right]^2}} .$$

Subtracting, we get from these equations

$$(H_0 - E_0) - (H_1 - E_1) = L\left\{\frac{1}{\sqrt{1 - \left[\frac{v}{V}\right]^2}} - 1\right\} .$$

The two differences of the form $H - E$ occurring in this expression have a simple physical meaning. H and E are the energy values of the same body, referred to two coordinate systems in relative motion, the body being at rest in one of the systems (system (x,y,z)). Hence it is clear that the difference $H - E$ can differ from the body's kinetic energy K with respect to the other system (system (ξ,η,ζ)) solely by an additive constant C, which depends on the choice of the arbitrary additive constants of the energies H and E. We can therefore put

$$H_0 - E_0 = K_0 + C$$

$$H_1 - E_1 = K_1 + C,$$

since C does not change during the emission of light. Thus, we get

$$K_0 - K_1 = L \left\{ \frac{1}{\sqrt{1 - \left(\frac{v}{V}\right)^2}} - 1 \right\} .$$

The kinetic energy of the body with respect to (ξ, η, ζ) decreases as a result of the emission of light by an amount that is independent of the body's characteristics. Furthermore, the difference $K_0 - K_1$ depends on the velocity exactly like the kinetic energy of the electron (*loc. cit.*, §10).

Neglecting quantities of the fourth and higher orders, we can put

[2] $$K_0 - K_1 = \frac{L}{V^2} \frac{v^2}{2}.$$

From this equation it follows directly:

If a body releases the energy L in the form of radiation, its mass decreases by L/V^2. Since obviously here it is inessential that the energy withdrawn from the body happens to turn into energy of radiation rather than into some other kind of energy, we are led to the more general conclusion:

The mass of a body is a measure of its energy content; if the energy changes by L, the mass changes in the same sense by $L/9 \cdot 10^{20}$, if the energy
[3] is measured in ergs and the mass in grams.

Perhaps it will prove possible to test this theory using bodies whose energy content is variable to a high degree (e.g., salts of radium).

If the theory agrees with the facts, then radiation transmits inertia
[4] between emitting and absorbing bodies.

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