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PHYSICS

COHERENT VISIBLE RADIATION OF FAST ELECTRONS PASSING THROUGH MATTER

By I. FRANK and Ig. TAMM, Corresponding Member of the Academy

In 1934 P. A. Čerenkov has discovered a peculiar phenomenon, whicd he has since investigated in detail (1). All liquids and solids if bombarded by fast electrons, such as β-electrons or Compton electrons produced by γ-rays, do emit a peculiar visible radiation, quite different from the eventual ordinary flourescence. This radiation is partially polarized, the electric oscillation vector being parallel to the electron beam, and its intensity can be reduced neither by temperature nor by addition to the liquid bombarded of quenching substances. The peculiarity of these caracteristics was scrutinized by Wawilow (2) who suggested that this radiation must be connected with the «Bremsung» of fast electrons. Since then a new and undoubtedly the most peculiar characteristic of the phenomenon was discovered, namely, its highly pronounced asymmetry, the intensity of light emitted in the direction of the motion of electrons being many times larger than in the backward direction. It follows that the substance bombarded radiates coherently for the space of at least one wavelength of the visible light.

This peculiar radiation can evidently not be explained by any common mechanism such as the interaction of the fast electron with individual atom or as radiative scattering of electrons on atomic nuclei* On the other hand, the phenomenon can be explained both qualitatively and quantitatively if one takes in account the fact that an electron moving in a medium does radiate light even if it is moving uniformly provided that its velocity is greater than the velocity of light in the medium.

We shall consider an electron moving with constant velocity v along the z axis through a medium characterized by its index of refraction n. The field of the electron may be considered as the result of superposition of spherical waves of retarded potential, which are being continually emitted by the moving electron and are propagated with the velocity $\frac{c}{n}$. It is easy to see that all these consecutive waves emitted

* The intensity of visible light emitted by the last named process is about 10^4 times smaller than the intensity observed.

will be in phase along the direction making the angle θ with the axis of motion z, if only v, n and θ do satisfy the condition

$$\frac{c}{n} = v \cos\theta; \quad \cos\theta = \frac{1}{\beta n}\,. \tag{1}$$

where $\beta = \frac{v}{c}$. Thus, there will be a radiation emitted in the direction θ, whereas the interference of waves will prevent radiation in any other direction. Now the condition (1) can be fulfilled only if $\beta n > 1$, i. e. only in case of fast electrons in a medium, whose index of refraction n for frequencies in question is markedly larger than 1. For instance, if $n = 1.33$ (water, $\lambda = 5900$ Å) the energy of the electron must be not smaller than 260 kV. But if $\beta n > 1$, then even an uniformly moving electron does radiate light in the direction θ *.

We proceed to develop a more detailed theory. Since we are intersted in visible radiation we can treat the medium macroscopically, applying to it the usual equations of the electromagnetic theory of light. Using the dinamical relation between the polarisation $\mathbf{P}$ and the electric intensity $\mathbf{E}$:

$$\frac{\partial^2 \mathbf{P}}{\partial t^2} + \sum_s \omega_s^2 \mathbf{P}_s = \alpha \mathbf{E},$$

where ω_s are frequencies of the molecular oscillators of the medium, and expanding all the field variables in Fourier series:

$$\mathbf{E} = \int_{-\infty}^{+\infty} \mathbf{E}_\omega e^{i\omega t} d\omega, \quad \mathbf{P} = \int_{-\infty}^{+\infty} \mathbf{P}_\omega e^{i\omega t} d\omega \quad \text{etc.}, \tag{2}$$

one easily obtains the connexion between $\mathbf{P}_\omega$ and $\mathbf{E}_\omega$:

$$\mathbf{P}_\omega = (n^2 - 1)\, \mathbf{E}_\omega\,, \tag{3}$$

where n is the refraction index of the medium for the frequency ω.

With the help of (2) and (3) one can easily reduce the Maxwells equations to the following set of equations:

$$\mathbf{H}_\omega = \operatorname{rot} \mathbf{A}_\omega, \quad \mathbf{E}_\omega = -\operatorname{grad} \varphi_\omega - \frac{i\omega}{c} \mathbf{A}_\omega = -\frac{ic}{\omega n^2} \nabla \operatorname{div} \mathbf{A}_\omega - \frac{i\omega}{c} \mathbf{A}_\omega, \tag{4}$$

$$\nabla^2 \mathbf{A}_\omega + \frac{\omega^2 n^2}{c^2} \mathbf{A}_\omega = -\frac{4\pi}{c} j_\omega\,, \tag{5}$$

where we made use of the connection between the vector and scalar potentials:

$$\operatorname{div} \mathbf{A}_\omega + \frac{i\omega}{c} n^2 \varphi_\omega = 0.$$

If an electron e is moving through the medium along the axis z with a constant velocity v, the corresponding current density j is equal to

$$j_x = j_y = 0, \quad j_z = e v \delta(x)\, \delta(y)\, \delta(z - vt),$$

where δ denotes the Dirac's function.

Expanding j_z one gets:

$$i_z(\omega) = \frac{e}{2\pi} e^{-\frac{i\omega z}{v}} \delta(x)\, \delta(y),$$

* X-rays can never be radiated by an uniformly moving electron since for these rays $n \leqslant 1$.

or, introducing cylindrical coordinates ρ, φ, z

$$j_z(\omega) = \frac{e}{4\pi^2\rho} e^{-\frac{i\omega z}{v}} \delta(\rho).$$

Introducing this expression in (5) and putting

$$A_\rho = A_\varphi = 0, \; A_z(\omega) = u(\rho) \cdot e^{-\frac{i\omega z}{v}}, \tag{6}$$

we obtain

$$\frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial u}{\partial \rho} + s^2 u = -\frac{e}{\pi c \rho}\delta(\rho), \tag{7}$$

where

$$s^2 = \frac{\omega^2}{v^2}(\beta^2 n^2 - 1) = -\sigma^2. \tag{8}$$

Thus u is a cylinder function satisfying the Bessel equation:

$$\frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial u}{\partial \rho} + s^2 u = 0, \tag{9}$$

everywhere with the exception of the pole $\rho = 0$. To find the condition to be satisfied by u at $\rho = 0$ we first replace the right hand side of (7) by f:

$$f = -\frac{2e}{\pi c \rho_0^2} \quad \text{if} \quad \rho < \rho_0, \quad f = 0 \quad \text{if} \quad \rho > \rho_0;$$

integrate then this equation over the surface of the circle of radius ρ_0 and lastly go over to the limit $\rho_0 \to 0$. In this way we obtain

$$\lim_{\rho \to 0} \rho \frac{\partial u}{\partial \rho} = -\frac{e}{\pi c}. \tag{10}$$

We have now to distinguish between two different cases. First consider the case of small velocities such that $\beta n < 1$, $s^2 < 0$ and $\sigma^2 = -s^2 > 0$, σ being thus a real quantity. In this case the solution of (9) satisfiying (10) and vanishing at the infinity is

$$u = \frac{ie}{2c} H_0^{(1)}(i\sigma\rho), \tag{11}$$

$H_0^{(1)}$ being the Hankel function of the first kind.

If $\sigma\rho \gg 1$ one can use the asymptotic value of $H_0^{(1)}$ and obtain according to (6) and (11):

$$A_z = \frac{e}{c}\int_{-\infty}^{+\infty} \frac{e^{-\sigma\rho + i\omega\left(t - \frac{z}{v}\right)}}{\sqrt{2\pi\sigma\rho}} d\omega, \quad \sigma\rho \gg 1.$$

Thus in case of small velocities the field of the electron decreases exponentially with ρ, so that there is no radiation at all.

If however the velocity of the electron is so large that within a certain frequency range $\beta n = \frac{v}{c} n(\omega)$ is greater than 1, then within this range the parameter s [equation (8)] is real and the general solution of the equations (7) and (9) represents in infinity a cylindrical wave. Requiring u in this case to represent an outgoing and not an ingoing wave, we obtain the following solution of (9), satisfying the condition (10) as well:

$$u = -\frac{ie}{2c} H_0^{(2)}(s\rho) \quad \text{if} \quad \omega > 0, \tag{12}$$

and a complex conjugate expression if $\omega < 0$, s being assumed to be positive. Using the asymptotic value of $H_0^{(2)}$ for $s\rho \gg 1$ one gets from (6) and (12):

$$A_z(\omega) = -\frac{e}{c\sqrt{2\pi s\rho}}\, e^{i\omega\left(t-\frac{z}{v}\right)-i\left(s\rho-\frac{3\pi}{4}\right)}, \quad \omega > 0.$$

With the help of (8) one can transform the exponent as follows:

$$A_z(\omega) = -\frac{e}{c\sqrt{2\pi s\rho}}\, e^{i\omega\left(t-\frac{z\cos\theta+\rho\sin\theta}{w}\right)+\frac{3}{4}\pi i}, \quad (\omega > 0) \tag{13}$$

where the angle θ is defined by (1) and $w = \frac{c}{n}$.

Thus, if $\beta n > 1$ a wave is propagated in infinity along the direction θ. The electric vector of the wave lies in the meridian plane (z, ρ).

Calculating the field intensity in the wave zone with the help of (4) one gets

$$\left.\begin{aligned} H_\varphi &= -\frac{a}{\sqrt{\rho}}\int \sqrt{s}\, d\omega \cos\chi, \\ E_\rho &= -\frac{a}{c\sqrt{\rho}}\int \frac{\sqrt{\beta^2 n^2 - 1}}{\beta^2 n^2 \sqrt{s}}\, \omega d\omega \cos\chi, \\ E_z &= \frac{n}{c\sqrt{\rho}}\int \left(1-\frac{1}{\beta^2 n^2}\right)\frac{\omega d\omega}{\sqrt{s}} \cos\chi, \end{aligned}\right\} \tag{14}$$

where $a = \frac{e}{c}\sqrt{\frac{2}{\pi}}$ and $\chi = \omega\left(t - \frac{z\cos\theta + \rho\sin\theta}{w}\right) + \frac{\pi}{4}$; all other components of **E** and **H** vanish. In distinction to (2) the integration in (14) as well as in all following integrals has to be extended only over p o s i t i v e values of ω and has to be restricted to the frequency range defined by $\beta n(\omega) \geqslant 1$.

The total energy W radiated by the electron through the surface of a cylinder of the length l (the axis of the cylinder coinciding with the line of motion of the electron) is equal to

$$W = 2\pi\rho l \int_{-\infty}^{+\infty} \frac{c}{4\pi}\left[\mathbf{EH}\right]_\rho dt.$$

With the help of the formula:

$$\int_{-\infty}^{+\infty} \cos(\omega t + \alpha)\cos(\omega' t + \beta)\, dt = \pi\delta(\omega - \omega')$$

we find

$$W = \frac{e^2 l}{c^2}\int_{(\beta n > 1)} \omega d\omega\left(1 - \frac{1}{\beta^2 n^2}\right). \tag{15}$$

We obtain exactly the same result if we calculate the total energy radiated by an electron,. which, being initially at rest, has moved with the velocity v through the distance l and was than stopped again. In this case the validity of (15) is restricted by the condition that l should be large in comparison with the wave length λ of the radiation emitted.

If the velocity of the electron is gradually decreasing, the equation (15) will thus remain valid for such sections l of its path, along which its velocity remains approximately constant, if only these sections are large enough in comparison with λ. Of course, in this case the angle θ between $\mathbf{v}$ and the direction of radiation will also gradually decrease.

In order to estimate the total loss of energy by radiation we can substitute in (15) for n^2 its approximate value defined by the equations

$$n^2(\omega) = 1 + \frac{A}{\omega_0^2 - \omega^2}, \quad n^2(0) = \varepsilon = 1 + \frac{A}{\omega_0^2},$$

where ε is the dielectric constant and ω_0 some average molecular frequency of the medium, and then integrate (15) from $\omega = 0$ to $\omega = \omega_0$.

In this way we obtain the following approximate expression for the loss of energy by radiation per unit path of the fast electron ($\beta \sim 1$):

$$\frac{dW}{dl} = \frac{e^2\omega_0^2}{2c^2}(\varepsilon - 1)\ln\frac{\varepsilon}{\varepsilon - 1}. \tag{16}$$

Assuming $\omega_0 = 6 \cdot 10^{15}$ sec.$^{-1}$, one finds $\frac{dW}{dl}$ to be of the order of several kilovolts per centimeter, a quantity negligible in comparison to the losses of energy by other causes.

When we have already finished our calculations, prof. A. Joffe kindly drew our attention to a paper of Sommerfeld ([4]) who had calculated the force acting on an electron moving in vacuum with a constant velocity $v > c$. This force is also due to radiation losses of the kind considered, but since the establishment of the theory of relativity we know that the condition $v > c$ can never be realized.

One easily deduces from the equation (15) that the number of photons emitted by an electron within the spectral region confined by the wave lengths λ_1 and λ_2 is equal to

$$N = 2\pi\alpha\left(\frac{l}{\lambda_2} - \frac{l}{\lambda_1}\right)\left(1 - \frac{1}{\beta^2 n^2}\right), \tag{17}$$

α being the fine structure constant, $\alpha = \frac{e^2}{\hbar c}$; and n, the average value of the refraction index in that region. Assuming $n = 1.33$, $\beta^2 = \frac{3}{4}$ (electron energy 500 kV) and $l = 0.1$ cm, we find that in the visible region between $\lambda_1 = 4 \cdot 10^{-5}$ cm and $\lambda_2 = 6 \cdot 10^{-5}$ cm about 10 photons are emitted by one fast electron. This agrees in order of magnitude with the experimental estimates (unpublished) of Čerenkov.

Čerenkov's measurement confirmed also the direct proportionality of the radiation intensity with the range l of the electrons in different mediums. The dependence of the intensity of radiation on the refraction index n is also discussed by him in detail in an article of the present fascicule; his conclusions are favourable to the theory.

If one takes in account, that Čerenkov's measurements were mostly made on widely diverging bundles of Compton electrons, produced by γ-rays and characterized by a very broad velocity distribution, one can safely say, that so far all the experimental evidence on the phenomenon in question, including the polarization and the spatial asymmetry of the radiation as well as its absolute intensity, is in best possible agreement with the theory here developed.

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